Estimating a Population Proportion

Example

What is the estimate of the proportion of households tuned to the super bowl?

Definitions

- Point Estimate = the value of a statistic that estimates the value of a parameter

Point Estimate for Population Proportion

$$\hat{p} = \frac{x}{n}$$

1. Find the point estimate

A sample was taken from students to ask them how many believe in BigFoot. Of the 300 surveyed, 25 believe in BigFoot. Find the point estimate of students who believe in BigFoot.

Definitions

- confidence interval = consists of an interval of numbers based on a point estimate
- level of confidence = represents the expected proportion of intervals that will contain the parameter if a larger number of different samples is obtained. The level of confidence is denoted $(1 - \alpha) \cdot 100\%$

Definitions

- margin of error: how much you could be off
- standard error: the standard deviation of the distribution of the sample proportion

Confidence Interval Estimates for the Population Proportion

Point estimate $\pm$ margin of error

Finding the Critical Value: $z_{\alpha/2}$

Case 1: You are given the % confidence level you desire (90% confidence interval)

1. Assign the percent to D (in decimal form)
2. Find: value = D + (1 − D) / 2
3. If on calculator: INVNORM(value)

   If using tables: Look up value in the

      middle

2. Find critical value that corresponds to the given level of confidence.

95%

Finding the Critical Value: $z_{\alpha/2}$

Case 2: You are given α

1. Find: value = 1 − (α / 2)
2. If on calculator: INVNORM(value)

   If using tables: Look up value in the

      middle

Sampling Distribution of $\hat{p}$

For a simple random sample of size n, the sampling distribution of $\hat{p}$ is approximately normal with mean $\mu_{\hat{p}} = p$ and standard deviation $\sigma_{\hat{p}} = \sqrt{\frac{p(1-p)}{n}}$, provided that np(1-p) ≥ 10. Note: Trials must be independent. Sample size can be no more than 5% of the population size (n ≤ 0.05N)

Constructing Confidence Interval for a Population Proportion

$$\text{Lower bound}: \hat{p} - z_{\alpha/2} \cdot \sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$$

$$\text{Upper bound}: \hat{p} + z_{\alpha/2} \cdot \sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$$

Finding Margin of Error When Constructing Confidence Interval for a Population Proportion

$$E = z_{\alpha/2} \cdot \sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$$

Formulas

$$\hat{p} = \frac{x}{n} \quad \hat{p} = \frac{LB + UB}{2} \quad E = UB - \hat{p}$$

3. Determine the point estimate of the population proportion, the margin of error and the number of individuals in the sample (x)

Lower bound: 0.20, upper bound 0.40, n = 200

4. Construct a confidence interval of the population proportion at the given level of confidence

x = 50, n = 200, 95% confidence

TI-83/84 Instructions

1. Stat
2. Tests
3. 1-PropZInt
4. Then enter x, n, and C-Level and then calculate

This will give you the interval. To find the margin of error (E): subtract the smallest part of your interval from $\hat{p}$

5. Confidence Interval

Given a survey of 1000 students, it was found that 250 of them enjoy zombie movies. Based on this sample, find the:

a) point estimate
b) 98% confidence interval of students who enjoy zombie movies
c) 95% confidence interval of students who enjoy zombie movies
d) margin of error of parts b and c

Finding Sample Size for Estimating Proportion p

$$\hat{p} \quad is \quad known: \quad n = \hat{p}(1-\hat{p})\left(\frac{z_{\alpha/2}}{E}\right)^2$$

$$\hat{p} \quad is \quad unknown: \quad n = 0.25\left(\frac{z_{\alpha/2}}{E}\right)^2$$

- Round Up if any Decimals
- E (margin of error) should always be in decimal form

6. Sample Size

We want to estimate the proportion of students that believe the moon walk was actually filmed in a movie studio. What size sample should be used if we want the estimate to be within 5% with 99% confidence if:

a) We use a prior estimate of 0.23 from last year
b) We do not use any prior estimate

7. Fun One

In a poll, 32% of people believe in UFO's. The margin of error in the poll was 2% and the estimate was made with 95% confidence. At least how many people were surveyed?