Form: $y' + P(x)y = Q(x)$

1. $y' + \underbrace{5}_{P(x)} y = x + e^{-3x}$

I.F.

② $\mu(x) = e^{\int 5\,dx}$

$\mu(x) = e^{5x}$

③ $e^{5x}(y' + 5y) = e^{5x}(x + e^{-3x})$

$\underbrace{e^{5x}}_{Q}\underbrace{y'}_{P'} + \underbrace{5e^{5x}}_{Q'}\underbrace{y}_{P} = xe^{5x} + e^{2x}$

$\int e^{5x}\,dx$

$u = 5x \quad du = 5\,dx$

④ $\frac{d}{dx}[y e^{5x}] = xe^{5x} + e^{2x}$

⑤ $\int \frac{d}{dx}[ye^{5x}]\,dx = \int (xe^{5x} + e^{2x})\,dx$

$ye^{5x} = \int (xe^{5x})\,dx + \int e^{2x}\,dx$

"PARTS" "u-sub"

| S | D | I |
|---|---|---|
| + | $x$ | $e^{5x}$ |
| − | $1$ | $\frac{1}{5}e^{5x}$ |
| + | $0$ | $\frac{1}{25}e^{5x}$ |

$ye^{5x} = \frac{1}{5}xe^{5x} - \frac{1}{25}e^{5x} + \frac{1}{2}e^{2x} + C$

$\frac{ye^{5x}}{e^{5x}} = \frac{1}{5}\cdot\frac{xe^{5x}}{e^{5x}} - \frac{1}{25}\cdot\frac{e^{5x}}{e^{5x}} + \frac{1}{2}\cdot\frac{e^{2x}}{e^{5x}} + \frac{C}{e^{5x}}$

$y = \frac{x}{5} - \frac{1}{25} + \frac{1}{2e^{3x}} + \frac{C}{e^{5x}}$

$\boxed{y = \frac{1}{5}x - \frac{1}{25} + \frac{1}{2}e^{-3x} + Ce^{-5x}}$