16. $\int \frac{1}{7\sqrt{5-x}\sqrt{x+2}}\,dx$

$u = \sqrt{x+2} = (x+2)^{\frac{1}{2}}$ $\quad du = \frac{1}{2}(x+2)^{-\frac{1}{2}}\,dx$

$u^2 = x+2$ $\quad du = \frac{1}{2(x+2)^{1/2}}\,dx$

$x = e - e^2$ $\quad du = \frac{1}{2\sqrt{x+2}}\,dx$

$= \frac{1}{7}\cdot 2\int \frac{\sqrt{x+2}}{2\sqrt{x+2}\sqrt{5-x}\sqrt{x+2}}\,dx$

$= \frac{2}{7}\int \frac{u}{\sqrt{5-(u^2-2)}\,u}\,du$

$= \frac{2}{7}\int \frac{1}{\sqrt{7-u^2}}\,du$

$= \frac{2}{7}\int \frac{1}{\sqrt{7-u^2}}\,du$

$= \frac{2}{7}\int \frac{1}{\sqrt{(\sqrt{7})^2-(u)^2}}\,du$

$a = \sqrt{7}$

$= \frac{2}{7}\int \frac{1}{\sqrt{a^2-u^2}}\,du$

$= \frac{2}{7}\arcsin\frac{u}{a} + C$

$= \boxed{\frac{2}{7}\arcsin\frac{\sqrt{x+2}}{\sqrt{7}} + C}$