1. $\int \frac{dx}{\sqrt{1-9x^2}}$

$= \int \frac{dx}{\sqrt{(1)^2-(3x)^2}}$

$a=1 \quad u=3x \quad du=3\,dx$

$= \frac{1}{3}\int \frac{3\,dx}{\sqrt{(1)^2-(3x)^2}}$

$= \frac{1}{3}\int \frac{du}{\sqrt{a^2-u^2}}$

$= \frac{1}{3}\arcsin\frac{u}{a}+C$

$= \frac{1}{3}\arcsin\frac{3x}{1}+C$

$= \boxed{\frac{1}{3}\arcsin(3x)+C}$

2. $\int \frac{1}{2+(x-4)^2}\,dx$

$= \int \frac{1}{(\sqrt{2})^2+(x-4)^2}\,dx$

$a=\sqrt{2} \quad u=x-4 \quad du=dx$

$= \int \frac{1}{a^2+u^2}\,du$

$= \frac{1}{a}\arctan\frac{u}{a}+C$

$= \boxed{\frac{1}{\sqrt{2}}\arctan\frac{x-4}{\sqrt{2}}+C}$

3. $\int \frac{1}{3x\sqrt{x^4-9}}\,dx$

$= \int \frac{1}{3x\sqrt{(x^2)^2-(3)^2}}\,dx$

$a=3 \quad u=x^2 \quad du=2x\,dx$

$= \frac{1}{3}\int \frac{1}{x\sqrt{(x^2)^2-(3)^2}}\,dx$

$= \frac{1}{3}\cdot\frac{1}{2}\int \frac{2x}{x\cdot x\sqrt{(x^2)^2-(3)^2}}\,dx$

$= \frac{1}{6}\int \frac{2x}{x^2\sqrt{(x^2)^2-(3)^2}}\,dx$

$= \frac{1}{6}\int \frac{1}{u\sqrt{u^2-a^2}}\,du$

$= \frac{1}{6}\cdot\frac{1}{a}\,\text{arcsec}\frac{|u|}{a}+C$

$= \frac{1}{6}\cdot\frac{1}{3}\,\text{arcsec}\frac{|x^2|}{3}+C$

$= \boxed{\frac{1}{18}\,\text{arcsec}\frac{x^2}{3}+C}$